STABILITY OF [illegible] SHIPS

IN A FOLLOWING SEA

by

David Lindley Folsom

Thesis Supervisor: Philip A. Mandel

Title: Professor of Naval Architecture

Date: 19 May 1967

# STABILITY OF AUTOPILOTED SHIPS IN A FOLLOWING SEA

BY

David Lindley Folsom

S.B., U.S. Coast Guard Academy

(1961)

Submitted in partial fullfillment of

the requirements for the degree

Master of Science

at the

Massachusetts Institute of

Technology

June 1967

Signature of Author ______________________________

Department of Naval Architecture     Date

and Marine Engineering

Certified By ______________________________

Thesis Supervisor

Accepted By ______________________________

Chairman, Departmental Comittee on

Graduate Students

STABILITY OF AUTOPILOTED SHIPS IN A FOLLOWING SEA

David Lindley Folsom

Submitted to the Department of Naval Architecture and Marine Engineering on 19 May 1967 in partial fulfillment of the requirements for the Master of Science Degree in Naval Architecture and Marine Engineering.

The object of this thesis was to investigate the stability of a ship with automatic steering in a following sea using the nonlinear equations of motion and to compare these results with the results of linear approximations.

The solution of the nonlinear equations was obtained by a technique of treating nonlinear terms as linear terms with non-constant coefficients. Results showed that when drift angles become 0.1 radians or greater the nonlinear solution differs substantially from the linear solution. But the nonlinear solution has a greater range of stability in the wave profile than the linear solution. This leads to the conclusion than a system designed using linear theory provides sufficient stability for the ship in a following sea.

The most important effect which this approach neglected was the coupling effect of the roll or heel motion on the yaw motion. Before a complete and accurate solution to the steering problem can be found the coupling effect must be investigated and some method of expressing it analytically must be produced.

Thesis Supervisor: Philip A. Mandel

Title: Professor of Naval Architecture

TABLE OF CONTENTS


| | Page |
|---|---|
| ABSTRACT | ii |
| INTRODUCTION | 1 |
| PROCEDURE | 3 |
| RESULTS | 13 |
| DISCUSSION OF RESULTS | 22 |
| CONCLUSIONS | 27 |
| RECOMMENDATION | 28 |
| APPENDIX A Bibliography | 30 |
| B Wave Forces and Moments | 32 |

# INTRODUCTION

Steering a ship in a following sea and the danger of the ship broaching in a following sea are problems which have been prominent for many years. Before the invention of the autopilot the only solution was a skilled helmsman and an experienced captain who avoided the situation whenever possible.

Steering in a following sea is not difficult until the speed of the ship becomes almost that of the waves. Then the forces and moments created by the waves on the ship are able to act for relatively long periods. In todays fast ships it is quite possible to encounter this situation of low frequency of encounter of ship to wave crest. There are two main solutions to the problem. The most obvious and by far the simplest to obtain is to reduce the speed of the ship which increases the frequency of encounter and shortens the period during which the forces act. But in todays world, time is money and to slow the ship in following seas is undesirable.

With an autopilot the steering problem can be treated analytically because the motion of the rudder is a function of the yaw and yaw velocity of the ship. The first explanations of the steering stability were made by using the linear terms of the equations of motion. However, in ship motions such as turning maneuvers the linear equations did not give adequate analytical results. Research has been done with the nonlinear equations of motion and solutions have been obtained using important nonlinear terms of the equations.These solutions were found to give adequate predictions of ship motion maneuvers.

In a following sea with small frequency of encounter the motions may become large and the linear solution may be very

inaccurate. Until now no one has tried to use the nonlinear equations to predict the stability of a ship traveling in a following sea.

The solution of the nonlinear equations of motion in the case of following seas is possible only after making several simplifying assumptions. The velocity of the ship is assumed to be the same as the wave thus the frequency of encounter is zero. The ship position in the wave profile is constant. The forces and moments created by the waves on the ship are determined by the Froude-Krylov Hypothesis. Cross coupling terms of roll and pitch motion are neglected in the equations. The surging motion of the ship is neglected. Having made these four assumptions it is now possible to solve the nonlinear equations of motion for a ship in a following sea.

## PROCEDURE

### NOMENCLATURE

The motions of the ship are based on axes through the center of gravity of the ship, Figure I, and the terms are defined below or in the reference material.

$\alpha$ = $\chi - \psi$ Heading angle from direction of advance of the waves

$\beta$ = Angle of drift or side slip

$\gamma$ = Coefficient of yaw feedback in rudder control system

$\lambda$ = Wave length

$\psi$ = Yaw angle or heading error

$\sigma$ = Coefficient of yaw velocity feedback in rudder control system

$a$ = Horizontal distance between wave crest and the center of gravity of the ship

$c_b$ = Block Coefficient

$\bar{y}$ = Lateral force exerted by the waves on the ship

$\bar{n}$ = Yawing moment exerted by waves on the ship

$\bar{h}$ = Wave amplitude

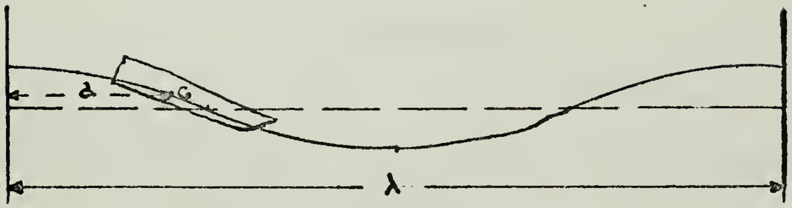
$\delta$ = Rudder angle

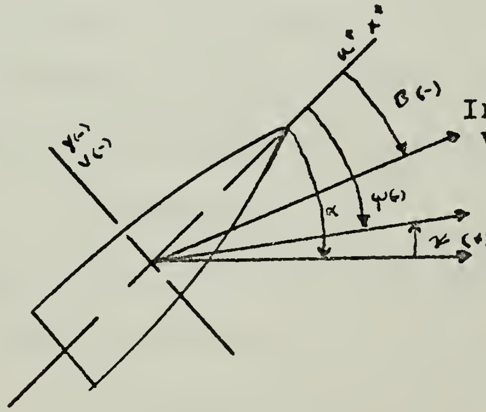
The derivatives and the nondimensionalizing factors for them are as defined in references (6) , (10) and (11).

Figure I

Definitions of Position, Distances

and Angles



## NONLINEAR EQUATIONS OF MOTION

The motions of the ship may be represented by nonlinear as derived in reference (10). These equations have been nondimensionalized in accordance with reference (11). The coefficients of the terms in the equations were determined for the Mariner hull model in reference (6). Using these equations and including only terms which reference (6) determined were important or of minor importance,the Y and N equations are expressed as

Y Equation

$$(m - Y_{\dot{v}})\dot{v} + (mx_G - Y_{\ddot{\psi}})\ddot{\psi} - Y_v v - 1/6\, Y_{vvv}v^3 - (Y_{\dot{\psi}} - mu)\dot{\psi} - 1/2\, Y_{\dot{\psi}vv}\dot{\psi}v^2 - Y_\delta \delta - 1/6\, Y_{\delta\delta\delta}\delta^3 - 1/2\, N_{\delta vv}\delta v^2 = 0 \tag{1}$$

N Equation

$$(mx_G - N_{\dot{v}})\dot{v} + (I_z - N_{\ddot{\psi}})\ddot{\psi} - N_v v - 1/6\, N_{vvv} v^3 - (N_{\dot{\psi}} - mx_G u)\dot{\psi} - 1/2\, N_{\dot{\psi}vv}\dot{\psi}v^2 - N_\delta \delta - 1/6\, N_{\delta\delta\delta}\delta^3 - 1/2\, N_{\delta vv}\delta v^2 = 0 \tag{2}$$

Substituting $\beta = -v$ into equations (1) and (2) equivalent equations are obtained using drift angle in lieu of lateral velocity. Adding to equations (1) and (2) the force and moment excitation of the waves as developed in Appendix B

Y Equation

$$(m - Y_{\dot{v}})\dot{\beta} + (mx_G - Y_{\ddot{\psi}})\ddot{\psi} - Y_v \beta - 1/6\, Y_{vvv}\beta^3 - (Y_{\dot{\psi}} - mu)\dot{\psi} - 1/2\, Y_{\dot{\psi}vv}\dot{\psi}\beta^2 - Y_\delta\delta - 1/6\, Y_{\delta\delta\delta}\delta^3 - 1/2\, Y_{\delta vv}\delta\beta^2 = F_{yy}\Phi \cos^2\gamma \sin(2\pi a/\lambda) - \psi\, \partial F_{yy}/\partial\gamma\, \Phi \cos^2\gamma \sin(2\pi a/\lambda) \tag{3}$$

N Equation

$$(mx_G - N_{\dot{v}})\dot{\beta} + (I_z - N_{\ddot{\psi}})\ddot{\psi} - N_v\beta - 1/6\, N_{vvv}\beta^3 - (N_{\dot{\psi}} - mx_G u)\dot{\psi} - 1/2\, N_{\dot{\psi}vv}\dot{\psi}\beta^2 - N_\delta\delta - 1/6\, N_{\delta\delta\delta}\delta^3 - 1/2\, N_{\delta vv}\delta\beta^2 = 1/2\, F_{xx}\Phi\cos^2\gamma\cos(2\pi a/\lambda) - \psi\, 1/2\, \partial F_{xx}/\partial\gamma\, \Phi\cos^2\gamma\cos(2\pi a/\lambda) \tag{4}$$

$$\Phi = k/C_b\, m' 4\pi^2 \bar{h}L/\lambda^2 \tag{5}$$

For a ship with an autopilot having yaw and yaw velocity feedback control the rudder equation is

$$\delta + \bar{t}\,\dot{\delta} = \gamma\psi + \sigma\dot{\psi} \tag{6}$$

## LINEAR SOLUTION OF EQUATIONS

The three equations in three unknowns are sufficient to give a solution. The difficulty arises in the fact that the equations are nonlinear differential equations and an analytical solution is not known. As a first approximation to the solution neglect all nonlinear terms and find the linear solution. This results in the following equations

Y Equation

$$\begin{aligned}(m - Y_{\dot{v}})\dot{\beta} + (mx_G - Y_{\ddot{\psi}})\ddot{\psi} - (Y_{\dot{\psi}} - mu)\dot{\psi} - Y_v\beta - Y_\delta\delta + \\ \partial F_{yy}/\partial\chi\,\Phi\cos^2\chi\sin(2\pi a/\lambda)\,\psi = F_{yy}\,\Phi\cos^2\chi\sin(2\pi a/\lambda)\end{aligned} \tag{7}$$

N Equation

$$\begin{aligned}(mx_G - N_{\dot{v}})\dot{\beta} + (I_z - N_{\ddot{\psi}})\ddot{\psi} - (N_{\dot{\psi}} - mx_G u)\dot{\psi} - N_v\beta - N_\delta\delta + \\ 1/2\,\partial F_{xx}/\partial\chi\,\Phi\cos^2\chi\cos(2\pi a/\lambda)\,\psi = 1/2\,F_{xx}\,\Phi\cos^2\chi \\ \cos(2\pi a/\lambda)\end{aligned} \tag{8}$$

Rudder Equation

$$\delta + \bar{t}\,\dot{\delta} = \gamma\psi + \sigma\dot{\psi} \tag{9}$$

Using letter designations for the coefficients and the values for the coefficients as determined in reference (6) the equations become

$$\begin{aligned}&A\dot{\beta} + C\ddot{\psi} + D\beta + E\dot{\psi} + G\delta + H\psi = I \\ &J\dot{\beta} + K\ddot{\psi} + L\beta + M\dot{\psi} + P\delta + R\psi = T \\ &\delta + \bar{t}\,\dot{\delta} = \gamma\psi + \sigma\dot{\psi}\end{aligned} \tag{10}$$

where the values of letter coefficients are given in Table I.

Table I

Values of Coefficients for Mariner Hull

| | | | | | |
|---|---|---|---|---|---|
| $A = (m - Y_{\dot{v}})$ | $= .01546$ | | $J = (mx_G - N_{\dot{v}})$ | $= .000227$ | |
| $C = (mx_G - Y_{\ddot{\psi}})$ | $= -.000086$ | | $K = (I_z - N_{\ddot{\psi}})$ | $= .00083$ | |
| $D = -Y_v$ | $= .011604$ | | $L = -N_v$ | $= .00264$ | |
| $E = -(Y_{\dot{\psi}} - mu)$ | $= .00499$ | | $M = -(N_{\dot{\psi}} - mx_G u)$ | $= .00166$ | |
| $G = -Y_\delta$ | $= -.002779$ | | $P = -N_\delta$ | $= .00138$ | |
| $H = \partial F_{yy}/\partial\chi\ \Xi\ \cos^2\chi\ \sin(2\pi a/\lambda)$ | | | $R = \frac{1}{2}\partial F_{xx}/\partial\chi\ \Xi\ \cos^2\chi\ \cos(2\pi a/\lambda)$ | | |
| $I = F_{yy}\ \Xi\ \cos^2\chi \sin(2\pi a/\lambda)$ | | | $T = \frac{1}{2} F_{xx}\ \Xi\ \cos^2\chi\ \cos(2\pi a/\lambda)$ | | |

The general solution of the simlutaneous equations

$$
\begin{aligned}
(As + D)\,\beta_i + (Cs^2 + Es + H)\,\psi_i + G\,\delta_i &= 0 \\
(Js + L)\,\beta_i + (Ks^2 + Ms + R)\,\psi_i + P\,\delta_i &= 0 \\
-(\sigma s + \gamma)\,\psi_i + (\bar{t}s + 1)\,\delta_i &= 0
\end{aligned}
\tag{11}
$$

where $\beta_i = \beta_1 e^{s_1 t} + \beta_2 e^{s_2 t} + \beta_3 e^{s_3 t} + \beta_4 e^{s_4 t}$

$\psi_i = \psi_1 e^{s_1 t} + \dots\dots\dots\dots\dots\dots$

$\delta = \delta_1 e^{s_1 t} + \dots\dots\dots\dots\dots\dots$

can be found by using determiants. The rrots of the solution are thus the stability roots of the system. The roots can be found by finding values of s for which the determinant is equal to zero.

CONTROLS FIXED STABILITY IN WAVES

For controls fixed stability there is no rudder term because the rudder deflection is zero. The controls fixed stability is determined from the following equation:

$$
\begin{aligned}
&(AK - JC)\,s^3 + (DK + AM - LC - JE)\,s^2 \\
&+ (DM + AR - EL - JH)\,s + (DR - LH) = 0
\end{aligned}
\tag{12}
$$

Substituting real values of coefficients from Table I into equation (12) the stability of the system is determined by use of Routh's Criterion for various positions of the ship in the wave profile and for various wave heights and wave lengths. Results are shown in Figures II and III.

STABILITY WITH AN AUTOPILOT IN WAVES

For Stability with an autopilot in waves the solution is determined from the following equation:

$$\begin{aligned}&(AK - JC)\bar{t}\, s^4 + ((KD + AM - JE - LC)\bar{t} + (AK - JC))\, s^3 \\ &+ ((DM + RA - JH - LE)\bar{t} + (KD + AM - JE - LC) + (AP - \\ &JG)\sigma)\, s^2 + ((RD - LH)\bar{t} + (DM + RA - JH - LE) + (PD - \\ &GL)\sigma + (AP - JG)\gamma)\, s + (RD - LH + (PD - GL)\gamma) = 0\end{aligned} \qquad (13)$$

Substituting real values of coefficients from Table I into equation (13) and assigning values of $\gamma + \sigma$ feedback coefficients the stability of the system can be found using Routh's Criterion. Results are shown in Figures IV, V and VI for the stability of the system in the wave profile for varying values of yaw feedback coefficient,with various wave heights and wave lengths.

In computing the data for Figures IV, V and VI it became obvious that yaw velocity feedback had no effect on the range of stability in the wave profile. Some method of showing the effect of yaw velocity feedback on the magnitude of stability was needed. This was done by assuming negligible time lag in the system. This makes one root very large and negative thus effectively reducing the equation to a cubic for which the solution of the important stability roots was obtained. In solving the cubic only the value of the largest stability root is determined. Reducing equation (13) to a cubic and solving,the effect of yaw velocity feedback on stability is as shown in Figure VII.

## NONLINEAR TERMS

Thus far the affect of yaw feedback coefficient and yaw velocity feedback coefficient has been shown using the linear equations of motion. Now some method of analytically expressing nonlinear terms must be developed in order to show their effect on the stability solution of the problem.

Going back to equations (3) and (4), the nonlinear terms of any importance are found to be the $\dot{\psi}\beta^2$, $\beta^3$ and $\delta\beta^2$ terms. Treating the terms as linear with nonconstant coefficients they can be expressed as follows:

$$Y_{vvv}\,\beta^3 \text{ becomes } (\beta^2\, Y_{vvv})\,\beta$$

$$Y_{\delta vv}\,\delta\beta^2 \text{ becomes } (\beta^2\, Y_{\delta vv})\,\delta$$

etc.

All the nonlinear terms can be treated in the above form. Putting these terms into equation (13) after assigning letter coefficients as shown in Table II the following determinental equation is produced:

$$\begin{aligned}
&(AK - JC)\bar{t}\, s^4 + ((KD + KD'\beta^2 + AM + AQ\beta^2 - JE \\
&-JF\beta^2 - LC - L'C\beta^2)\bar{t} + (AK - JC))\, s^3 + ((DM + D'M\beta^2 \\
&+ DQ\beta^2 + D'Q\beta^4 + RA - JH - LE - L'E\beta^2 - LF\beta^2 \\
&-L'F\beta^4)\bar{t} + (KD + KD'\beta^2 + AM + AQ\beta^2 - JE - JF\beta^2 \\
&- LC - L'C\beta^2) + (AP + AP'\beta^2 - JG - JG'\beta^2)\sigma)\, s^2 \qquad (14) \\
&+ ((RD + RD'\beta^2 - LH - L'H\beta^2)\bar{t} + (DM + D'M\beta^2 + DQ\beta^2 \\
&+ D'Q\beta^4 + RA - JH - LE - L'E\beta^2 - LF\beta^2 - L'F\beta^4) + (AP \\
&+ AP'\beta^2 - JG - JG'\beta^2)\gamma + (PD + P'D\beta^2 + PD'\beta^2 + \\
&P'D'\beta^4 - GL - GL'\beta^2 - G'L\beta^2 - G'L'\beta^4)\sigma)\, s + ((RD \\
&+ RD'\beta^2 - LH - L'H\beta^2) + (PD + P'D\beta^2 + PD'\beta^2 + \\
&P'D'\beta^4 - GL - GL'\beta^2 - G'L\beta^2 - G'L'\beta^4)\gamma) = 0
\end{aligned}$$

Table II

Coefficients of Nonlinear Terms for

Mariner Hull

| | | | |
|---|---|---|---|
| $F = -1/2\ Y_{\dot{\psi} vv}$ | $= -.0765$ | $Q = -1/2\ N_{\dot{\psi} vv}$ | $= .0274$ |
| $D' = -Y_{vvv}$ | $= .0808$ | $L' = -N_{vvv}$ | $= -.0164$ |
| $G' = -Y_{\delta vv}$ | $= .0119$ | $P' = -N_{\delta vv}$ | $= .00489$ |

## STABILITY WITH AUTOPILOT IN WAVES INCLUDING NONLINEAR TERMS

Substituting values from Tables I and II into equation (14) and by choosing an instantaneous value of drift angle, $\beta$, a solution for the stability can be determined for that instant of time. By choosing a value of drift angle which is felt to be the largest expected value, the solution for the roots is obtained. Plotting this solution along with the linear solution a range in which the true solution lies is found. These results are plotted in Figures VIII and IX.

From all of the previous equations it is now possible to investigate the steering stability of any ship in waves and to determine what steering characteristics are necessary to have directional stability in a following sea at any given sea condition.

## STABILITY FOR MARINER HULL IN REAL SEA CONDITIONS

In order to obtain information necessary to design a steering system that will ensure directional stability at all times the effects of time lag, feedback coefficients and rudder area on the stability must be known. There may be limitations on the values of yaw feedback coefficient and yaw velocity feedback coefficient because operation in head seas and calm water must be part of the design. These operations may require some range of values of $\gamma$ and $\sigma$. Time lag does not appear to be a critical parameter. Present

system time lags are in the area of 0.1 and this is sufficiently small for stability in following seas.

The first consideration is that the rudder area must be sufficient to counteract the moment created by the waves. For the Mariner hull a check for sufficient rudder area is made by equating the moment created by the waves to the moment which can be applied by the rudder. Figure X shows the angle needed to overcome moment of waves alone. The present Mariner rudder appears to be large enough to counteract wave moment.

In all of the developments using the Mariner hull the assumption is made that the ship is somehow able to maintain the same speed as the waves. Thus the problem as developed represents a more dangerous condition than is actually present.

The next step is to find the value of yaw angle feedback coefficient necessary to make the system stable in design sea conditions. Using equation (13) and the values of Tables I and II the feedback coefficient necessary for stability is found. Results are shown in Figure XI.

Another question that arises is how would changing the rudder area change the stability feedback coefficients. This is done by using equation (13) and varying the rudder force and moment derivatives. Using the rudder coefficient changes of reference (9) as a basis a direct percentage change in Mariner coefficients is made in order to obtain an estimate of how Mariner coefficients would change with the same rudder area changes. The effect of rudder area changes on angle to overcome wave moment are shown in Figure XII. The effects of rudder area changes on yaw angle feedback coefficient are shown in Figure XIII.

From the results of these investigations in following seas

and from investigations in head seas and calm water maneuvers the best combination of rudder area, time lag, and steering system feedback coefficients can be chosen.

## RESULTS

The following figures show how the directional stability of a ship is affected by the presence of waves when running in a following sea.

Figures II and III show how the range of instability in the wave profile for the ship with no steering control for varying wave height and wave length.

Figures IV, V and VI show how the range of instability can be decreased and even eliminated by the addition of automatic steering controls with yaw angle feedback control.

Figure VII shows how the real and imaginary part of the largest stability root varies in the wave profile for a system which is stable all the time for two values of yaw velocity feedback coefficient.

Figures VIII and IX show how the nonlinear terms in the equations of motion affect the largest stability root and the range of stability in the wave profile.

Figure X shows the rudder angle necessary to counteract the constant wave moment acting on the Mariner hull.

Figure XI shows the minimum required yaw angle feedback coefficient ncessary for a stable system as a function of the sea condition.

Figure XII and XIII show how the rudder angle reuired to counteract wave moment and the required yaw angle feedback coefficient are changed when rudder area is changed.

Figure XIV shows how the minimum required yaw angle feedback coefficient varies as the size of the ship changes for operation in the same sea conditions.

Figure II

Stability of Controls Fixed Ship in Waves

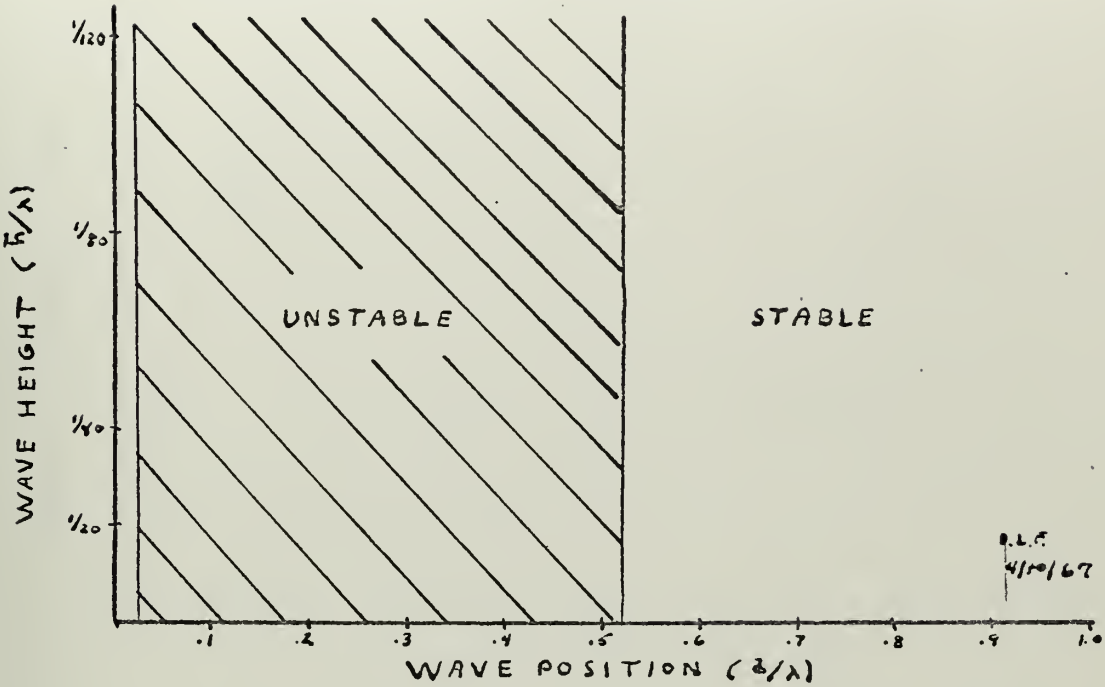



Figure III

Stability of Controls Fixed Ship in Waves

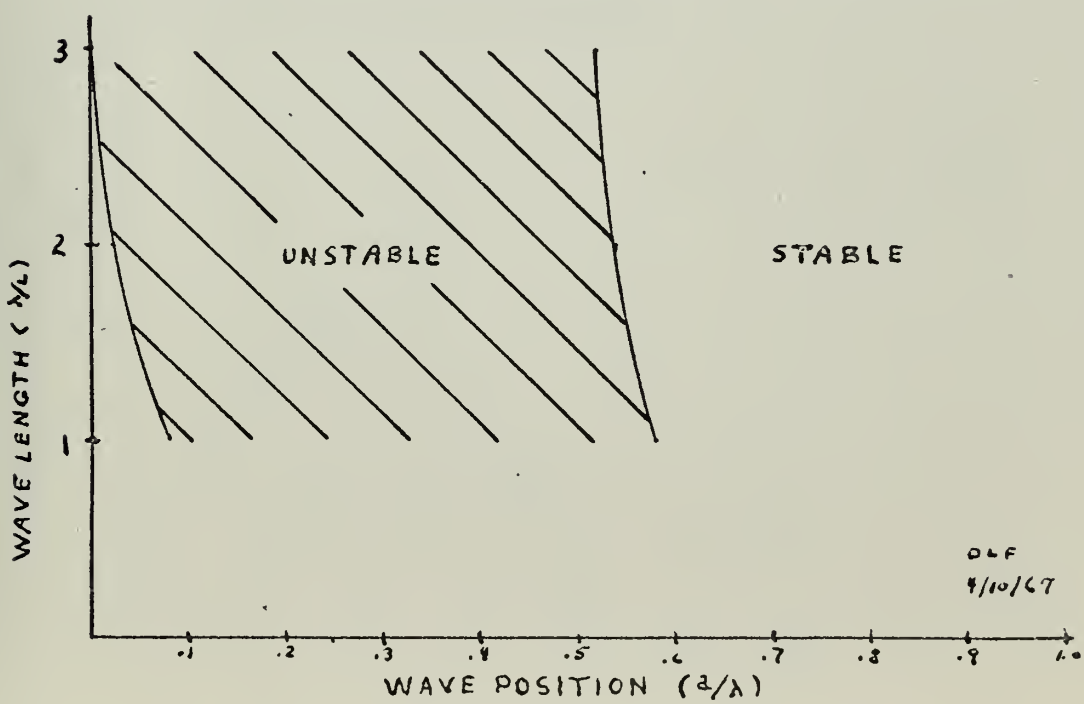


Figure IV

Stability of Autopiloted Ship with Yaw Angle Feedback Control

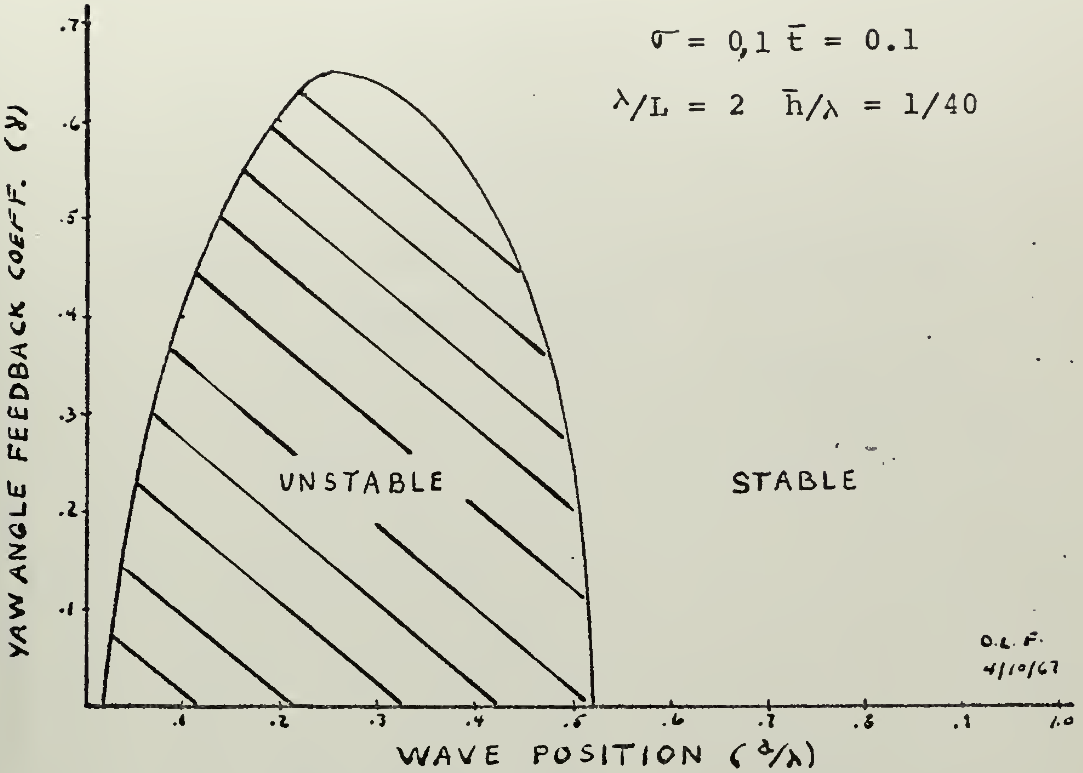



Figure V

Stability of Autopiloted Ship with Yaw Angle Feedback Control

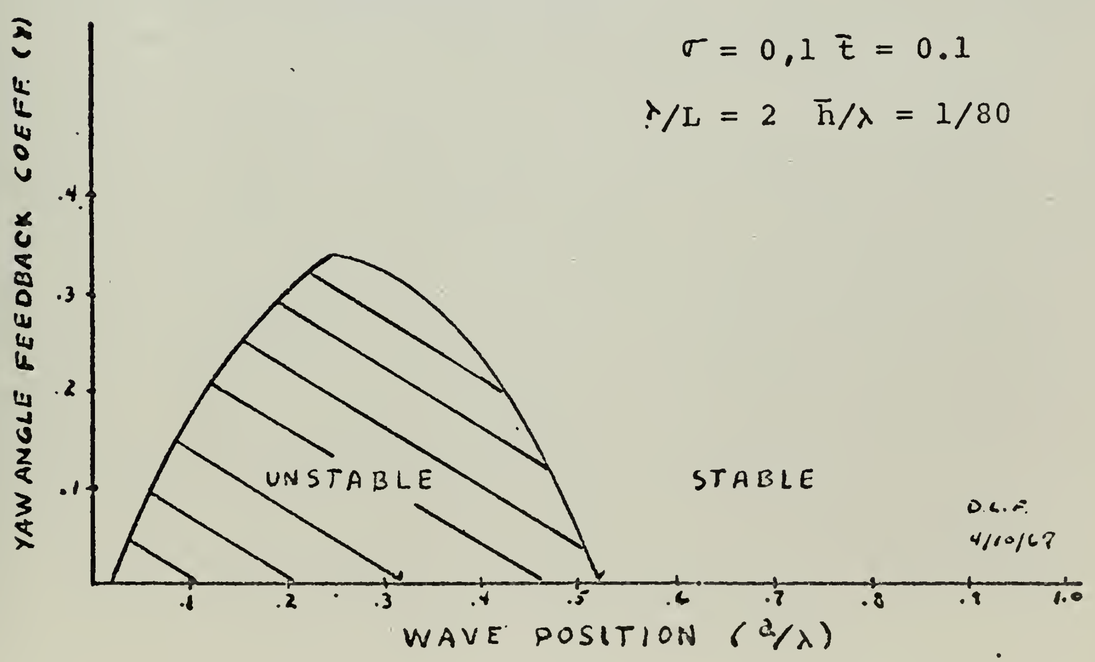


Figure VI

Stability of Autopiloted Ship with Yaw Angle

Feedback Control

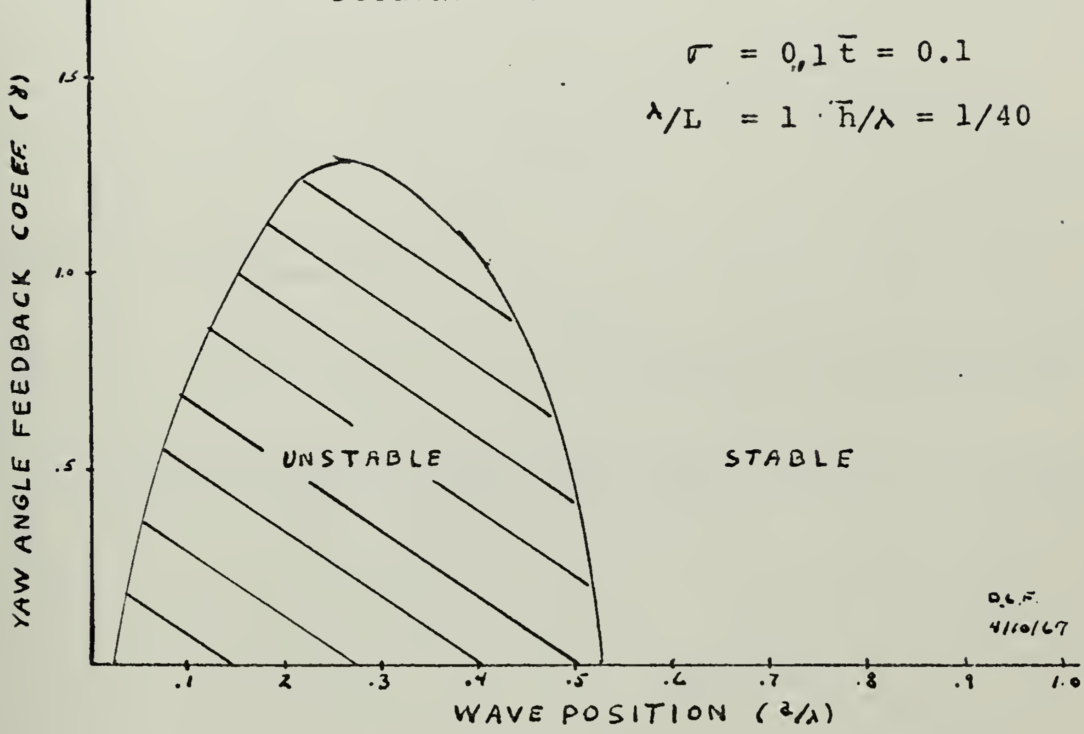


Figure VII

Real and Imaginary Parts of Largest Stability Root

$\lambda/L = 2 \quad \bar{h}/\lambda = 1/40$

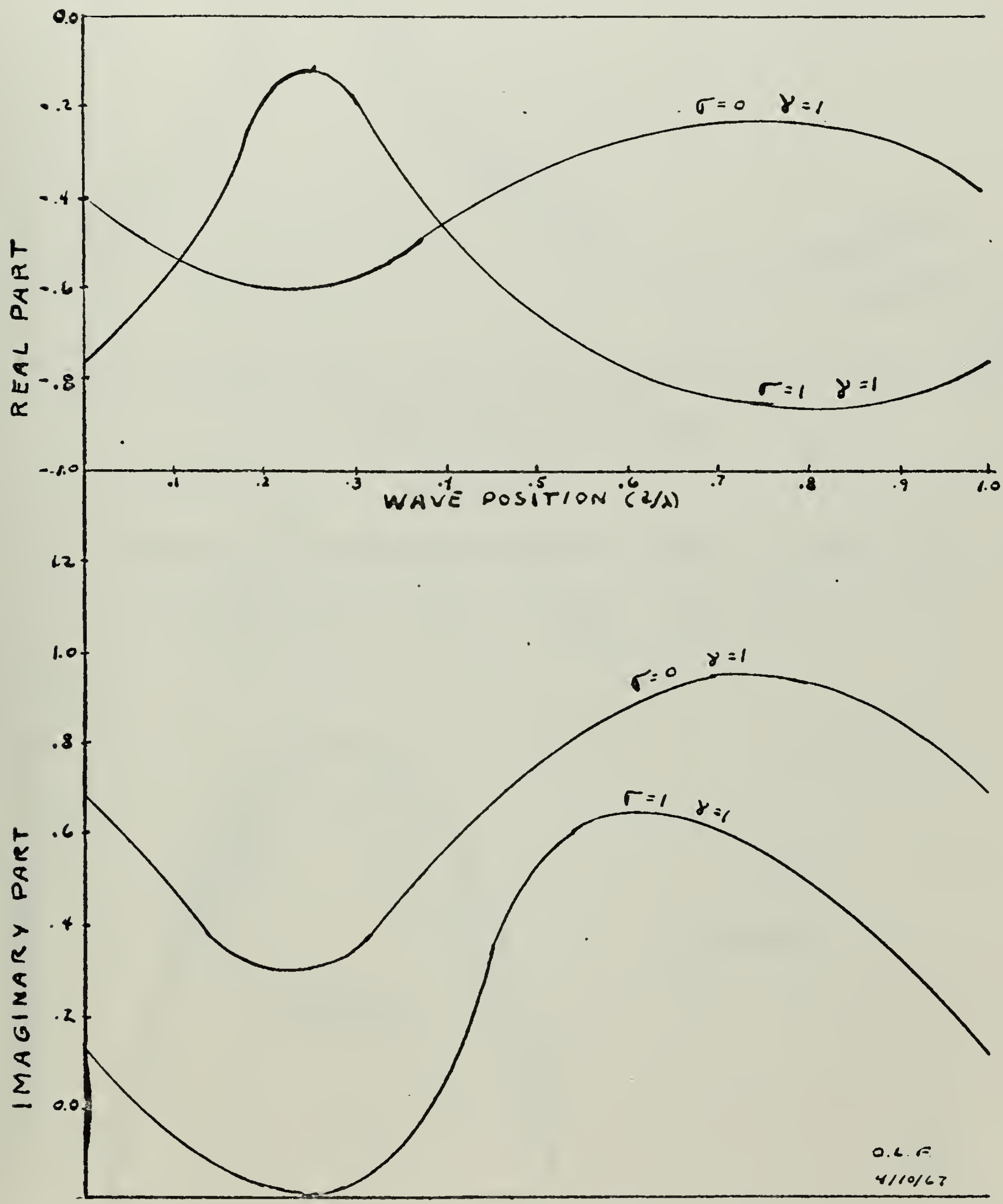


Figure VIII

Real Part of the Largest Stability Root

$\sigma = 1 \quad \gamma = 1 \quad \lambda/L = 2 \quad \bar{h}/\lambda = 1/40 \quad \beta = .1$

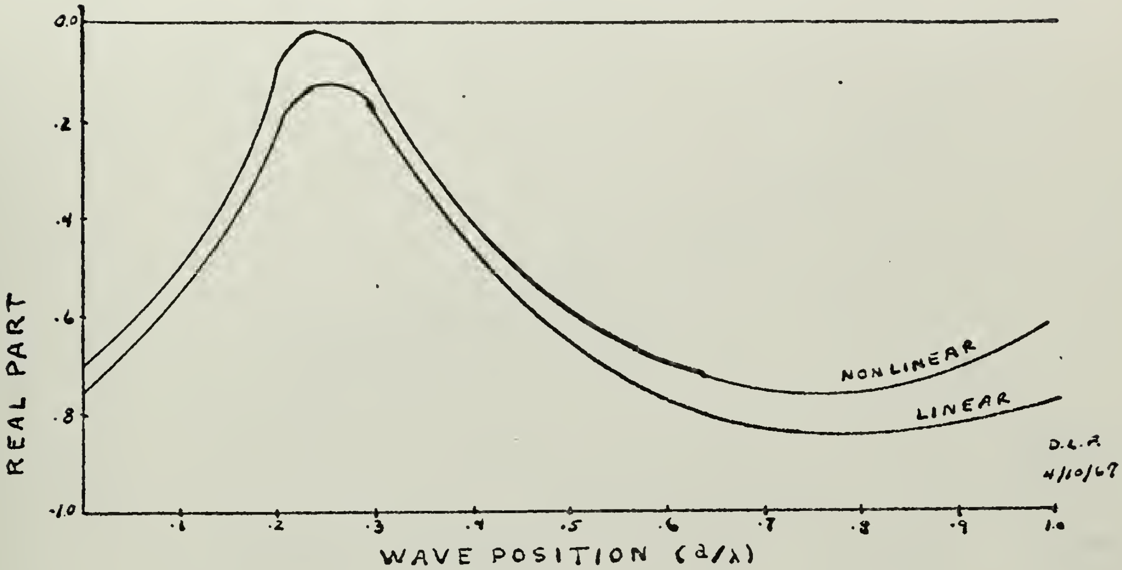



Figure IX

Effect of Nonlinear Terms on Range of Stability

$\sigma = 0 \quad \bar{\varepsilon} = .1 \quad \lambda/L = 1 \quad \bar{h}/\lambda = 1/40 \quad \beta = 0.1$

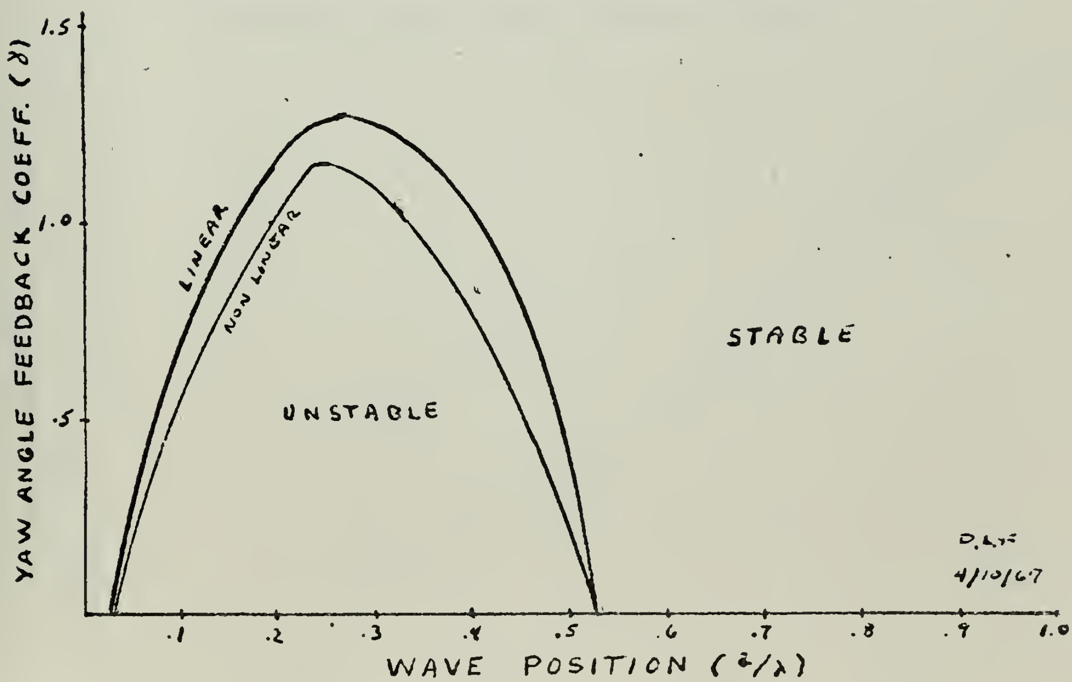


Figure X

Rudder Angle Needed to Overcome Wave Moment

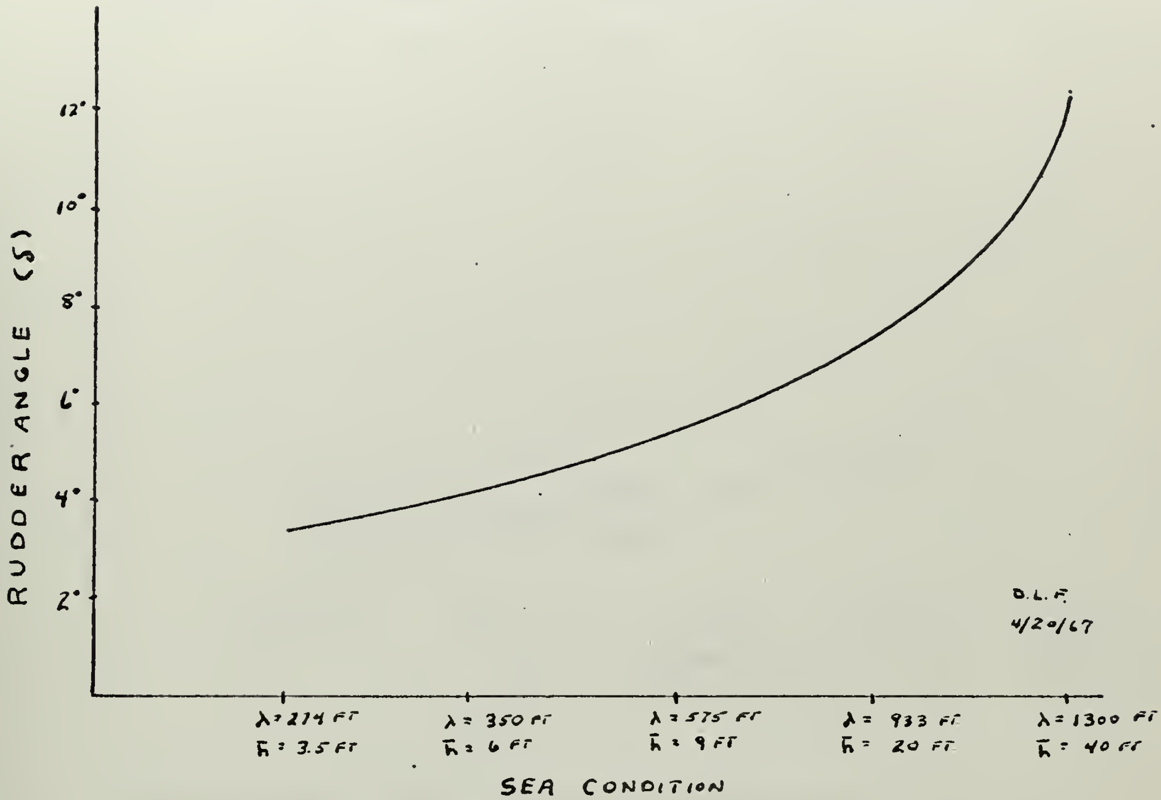



Figure XI

Yaw Angle Feedback Coefficient Necessary for

for Stability Using Linear Theory

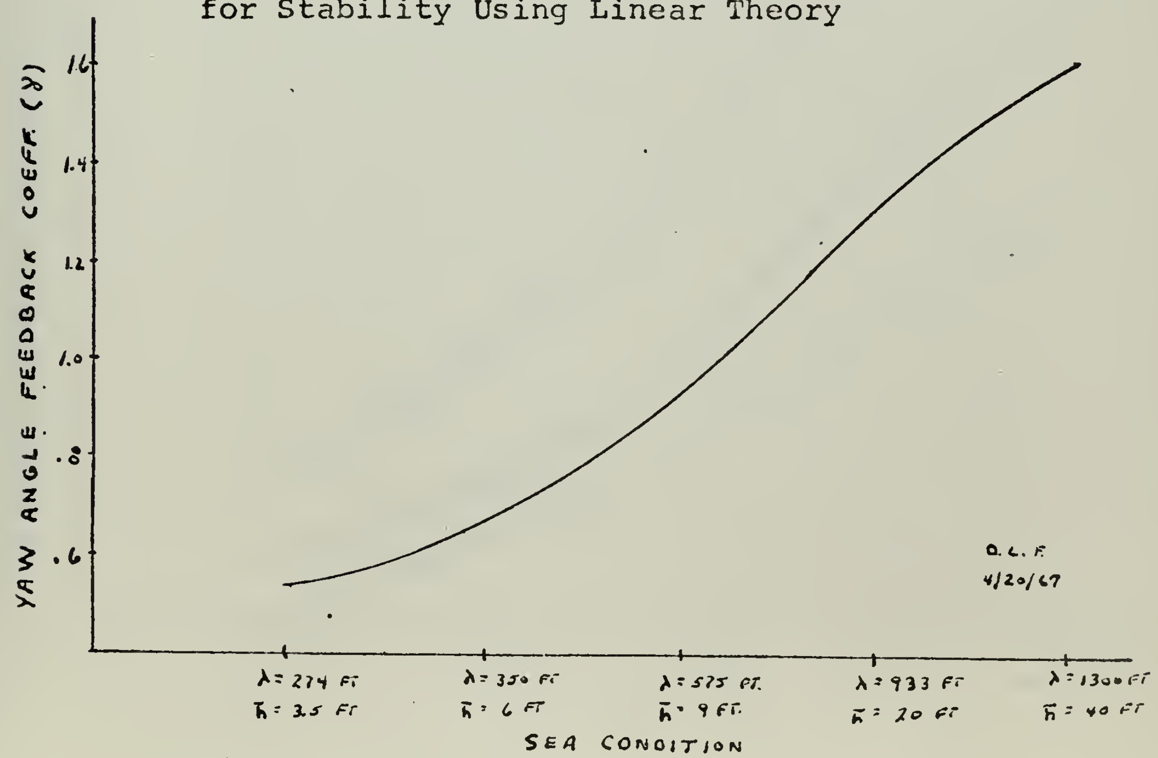


Figure XII

Rudder Angle Required to Counteract Wave Moment

for Different Rudder Areas

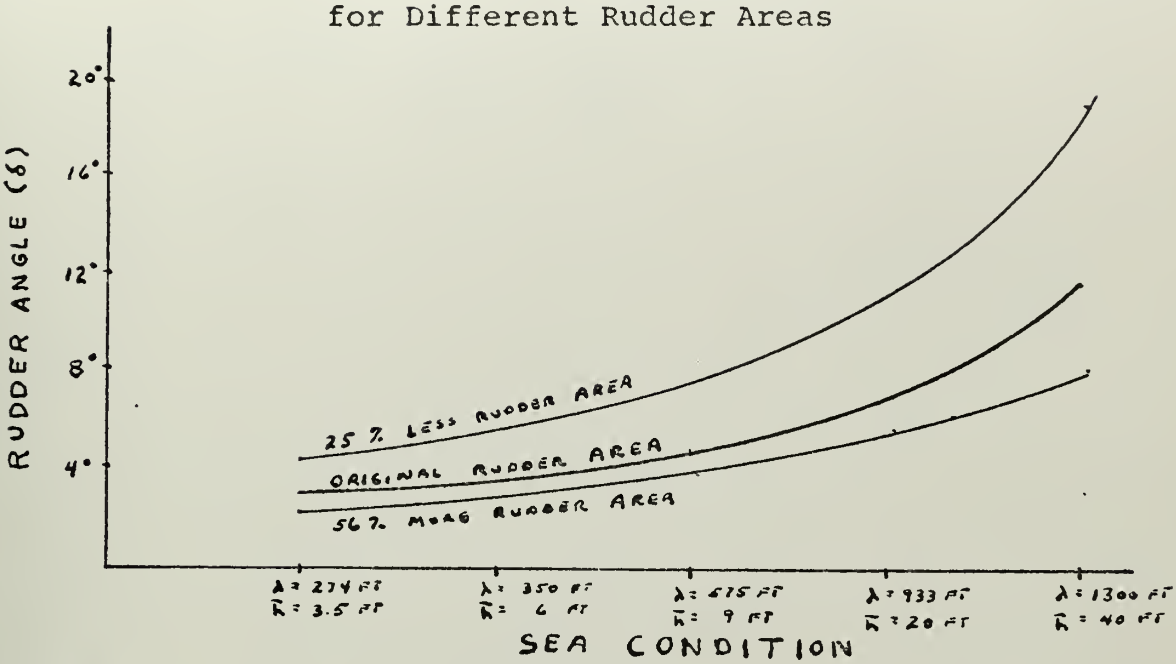



Figure XIII

Required Yaw Angle Feedback Coefficient for

Different Rudder Areas

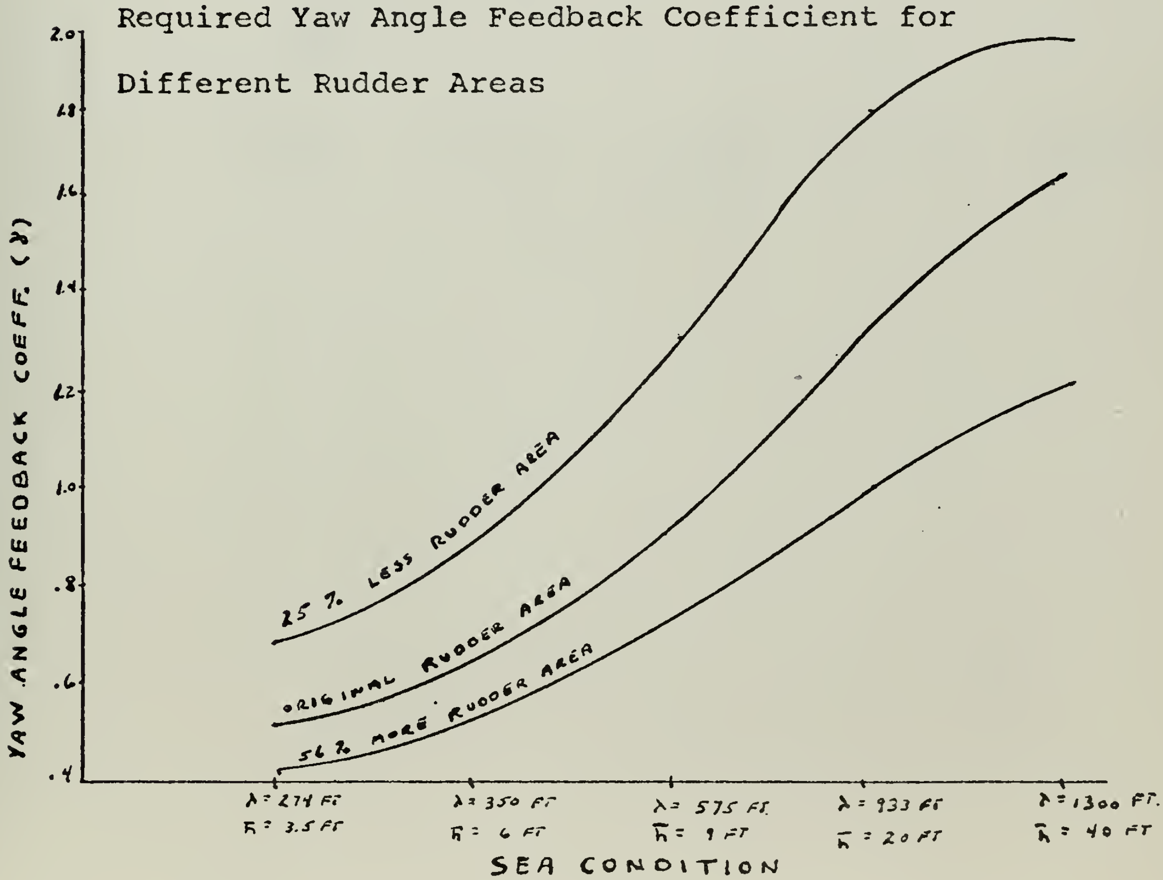
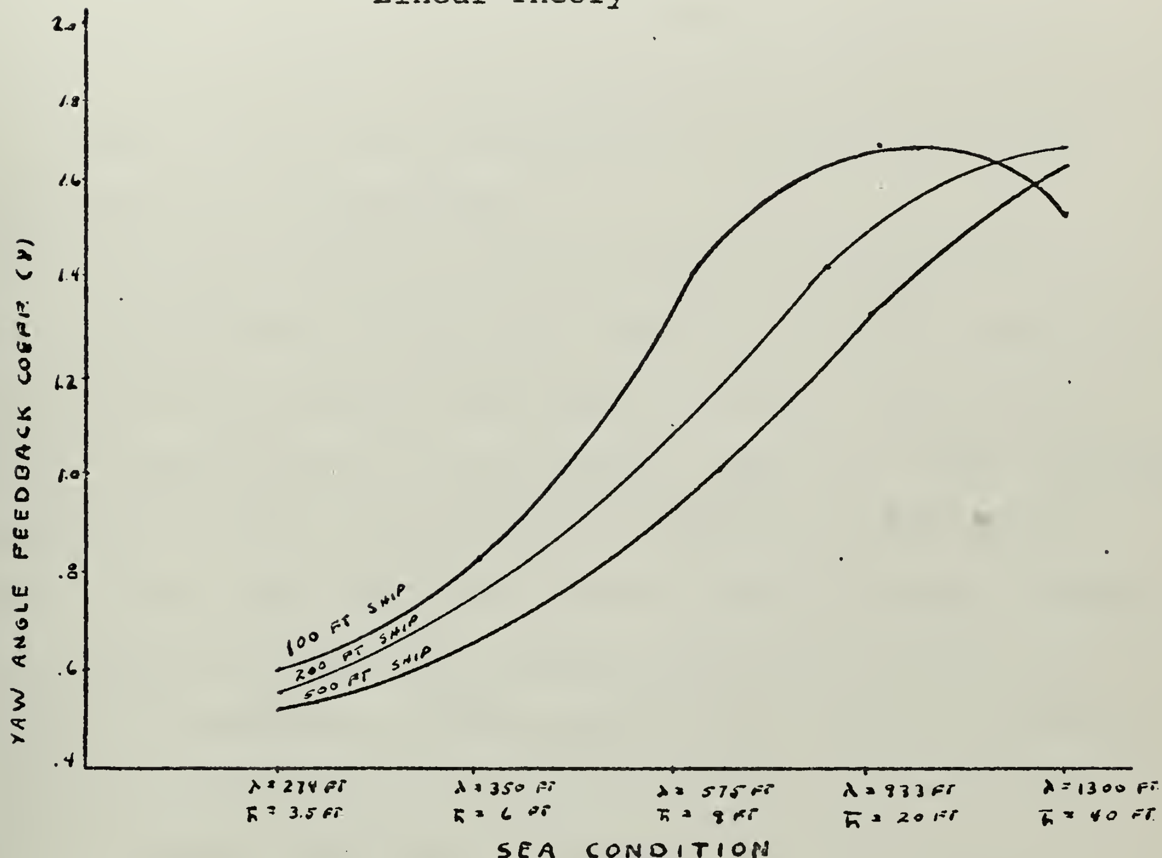


Figure XIV

Yaw Angle Feedback Coefficient Required for

Stability for Various Ship Lengths Using

Linear Theory



## DISCUSSION OF RESULTS

### NONLINEAR EQUATIONS

The solution of the nonlinear equations presented an impossible task. To date the only solutions to the equations have been by numerical analysis which was designed to predict the motion of the ship during standard maneuvers in calm water. To obtain a numerical solution for steering in following seas was entirely beyond my comprehension.

The only path left open was to try and find some method of obtaining an approximate analytical solution. Previous analytical solutions had been obtained using only the linear terms of the equations. Investigation of the important nonlinear terms resulted in the discovery that they were all a function of drift angle squared or drift angle cubed. By treating the drift angle squared as a non constant coefficient equations evolved as linear with non constant coefficients. These equations were solved by choosing a maximum expected drift angle and putting it into the equations as a constant. This gave a stability solution for an instantaneous value of drift angle. The maximum expected drift angle solution is then the solution for largest ship motions.

Comparing the linear solution and nonlinear solution of the largest stability root,Figure VIII, the nonlinear solution shows less stability than the linear solution. Figure IX shows that the ship has a greater range of stability in the wave profile than predicted by the linear solution. These results correlate with the nonlinear solutions of the standard maneuvers. In the standard maneuvers of reference (9) the linear solution predicts less stability and thus better turning characteristics than is actually present. The nonlinear terms represent forces and moments

which are always opposing the yawing motion. As a result the range of stability in the wave profile is actually greater than the linear prediction but the oscillating motion of the system does not decay as rapidly as linear theory predicts.

The actual stability of the system lies somewhere in between the linear prediction and the nonlinear prediction because the drift angle is always changing as the ship moves through the waves. The graphs are plotted for a drift angle of six degrees. In the turning circle maneuver the maximum drift angle was nine degrees and for the zig zag maneuver the drift angle reached seven degrees.

In designing a steering system the use of the linear theory will provide more than adequate control and so the design is sufficient for all conditions.

## DESIGN OF STEERING CONTROLS

In the design of an automatic steering system the rudder size, steering equipment size and the feedback to the control circuits are the parameters which must be determined. It is very important to be able to see what effect these parameters have on the steering problem in a following sea.

The rudder size and the steering gear size are initially determined by the requirements set for the ship to make the standard maneuvers in calm water.

With this rudder size Figures II and III show the stability region for the ship if it has no controls and the rudder remains fixed at zero deflection. The ship is unstable on the negative slope of the wave and stable on the positive slope. This stability region is unchanged by wave height but does shift with varying wave length because the wave slope changes.

Adding automatic controls to the ship with its present rudder, Figures IV, V and VI show how the value of yaw angle feedback coefficient required for stability varies for different types of following seas. The value of yaw angle feedback coefficient needed for stability is very dependent upon the size and length of the waves. As wave height increases the more yaw angle feedback is needed to make the system stable. The important wave lengths are in the tange 1.0 to 8.0 of ship length. Shorter than this and the wave has little affect on steering. Longer waves require the same or less feedback due to decreasing slope of the wave.

Yaw angle feedback is the controling feedback for stability. The effect of adding yaw velocity feedback to the system is shown in Figure VII. At zero frequency of encounter with the waves it has an unstabilizing affect. But this is for our own hypothetical situation. When the frequency of encounter is not zero but has a value of 0.5 radians/sec. or larger then L.J. Rydill,reference (8) has shown that velocity feedback helps stability. So in an actual following sea situation a value of yaw angle velocity feedback is important for good ship control.

Taking the Mariner hull in a following sea situation the effect of wave forces on stability can be shown. The effect of various sea states on a 500 foot Mariner is shown in Figures X and XI. Figure XI shows the minimum value of yaw angle feedback coefficient for operation in different sea states. Figures XII and XIII show how the minimum value of yaw angle feedback coefficient is changed by changing the rudder area. Figure XIV show how the required yaw angle feedback coefficient differs for a 200 foot and a 100 foot Mariner as compared to the 500 foot ship.

Having obtained the data for required yaw angle feedback coefficient with rudder area variation,this must be added to data generated for steering in head seas and in standard maneuvers to give a true picture of the limitations of the steering parameters in the different modes of operation. With this data one should be able to design the best possible steering system.

The results for the real ship case are highly dependent on available wave spectrum data. The data used here was for fully developed ocean waves as defined by the U.S. Hydrographic Office. In coastal waters it is very possible that the wave height to wave length ratio will be much larger for given wave lengths due to shallow water effects on the waves. This fact would require much larger values of yaw angle feedback coefficient to stabilize the ship. In designing a steering system it is most important that accurate data on operating sea conditions be used.

LIMITATIONS OF DESIGN PROCEDURE

There are several conditions and effects which may be of importance. that are not accounted for in this investigation. The most important is the cross coupling effect of the roll motion on the yawing motion. No one has yet been able to express analytically the effect of rolling motions on the steering problem. It is easy to see that when the ship rolls or heels during yawing motion that it becomes unsymetrical about the reference centerline and side forces and yawing moments are created. The larger the roll or heel angle the greater the forces and moments. This effect is greatly dependent on the static stability of the transverse motion. Investigation by the Germans, reference (12),in capsizing model tests shows that as the static stability of the ship is reduced it becomes less stable in yawing motions until it reaches a point

where the model broaches and eventually capsizes. From their investigation the importance of the yaw-roll coupling is shown. Until someone develops a method to express this effect a complete solution to the steering problem is not really known.

A condition which has not been investigated is the condition where the frequency of encounter with the waves is such that the wave moments are exerted when the ship is yawing away from the prescribed course such that the motion appears as a spring mass system being excited at half frequencies of its natural frequency. It seems possible that the ship yawing motion may somehow be expressed as a simple spring-mass-damper system with some natural frequency. Then the system could be checked at the half frequencies to see if the motion is stable or not. By changing the ammount of feedback the natural frequency would be changed and data could be obtained which may place further limitations on the values of the feedback coefficients.

Another effect which has been neglected is that of the wind force and moment on the ship. There may, in a sufficiently strong wind, be forces large enough to affect the steering problem.

The effect of the surging motion of the ship in a following sea is another condition which may have important adverse affects on the steering problem. For this investigation the surging motion was neglected in order to simplify the mathmatics of the problem. The surging may cause sufficient changes in forces and moments so as to cause instability.

## CONCLUSIONS

1. The nonlinear equations of motion will provide a more accurate solution when operating in a following sea because drift angles can be expected to be large.

2. The nonlinear solutions show that when designing a steering system use of the linear analysis will provide more stability than is actually required for operation in a following sea.

3. At very low frequencies of encounter the yaw angle feedback in the only control needed for stability. Yaw velocity feedback at low frequencies has an unstabilizing effect but is important for good stability when the frequency of encounter is above 0.5 radians/ sec..

4. Values of feedback coefficients can be determined in order to make the system stable when operating in following seas.

5. Values of feedback coefficient are highly dependent on the wave spectrum in which the ship is designed to operate.

6. The roll motion influence on the yaw motion can be very large for ships with low tranverse stability.

7. The point where wave excitation is at half frequency of yaw motion natural frequency may be as dangerous a position as the zero frequency of encounter point.

## RECOMMENDATIONS

There are three areas of the problem which need further investigation.

1. Explanation of the Roll-Yaw coupling effect which will produce the greatest changes in the yaw motion problem.

2. Investigation of half frequency excitation which may also produce some startling results.

3. Investigation of the surging motion effects which may cause slight changes in the problem.

It may also be possible to somehow produce the ship motions using an analog computer. If this can be done then the nonlinear solutions can be produced exactly and the answer to the half frequency problem could be solved at the same time.

# APPENDIX

## APPENDIX A

## BIBLIOGRAPHY


1. R. Wahab and W.A. Swaan

   Cousekeeping and Broaching of Ships in Following Seas

   Journal of Ship Research Volume 7 No. 4 April 1964

2. P. DuCane and G.J. Goodrich

   The Following Sea, Broaching and Surging

   The Royal Institution of Naval Architects September 1961

3. E.V. Lewis

   Ship Motions in Oblique Seas

   The Society of Naval Architects and Marine Engineers

   November 1960

4. E.V. Lewis

   Ship Speed in Irregular Seas

   The Society of Naval Architects and Marine Engineers

   November 1955

5. P. Mandel

   Subcritcal and Supercritical Operation of Ships in Waves

   and the Coincidence of Maximum Damping

   Journal of Ship Research Volume 4 No. 1 June 1960

6. J. Strom-Tejsen

   A Digital Computer Technique for Prediction of Standard

   Maneuvers of Surface Ships

   David Taylor Model Basin Report No. 2130 December 1965

7. G. Weinblum and M. St. Denis

   On the Motions of Ships at Sea

   The Society of Naval Architects and Marine Engineers

   Transactions 1950

8. L.J. Rydill

A Linear Theory for the Steered Motion of Ships in Waves

The Institution of Naval Architects August 1958

9. H. Eda and C.L. Crane

Steering Characteristics of Ships in Calm Water and Waves

The Society of Naval Architects and Marine Engineers

November 1965

10. M.A. Abkowitz

Lectures on Ship Hydrodynamics-Steering and Manoeuvrability

Hydro- and Aerodynamics Laboratory, Lyngby, Denmark

Report No. Hy-5 May 1964

11. Nomenclature for Treating the Motion of a Submerged Body Through a Fluid

The Society of Naval Architects and Marine Engineers

Technical and Research Bulletin No. 1-5

12. S. Kastner

Capsizing Tests With a Model in a Naturally Disturbed Sea

Bureau of Ships Translation No. 847

## APPENDIX B

## LATERAL FORCE AND MOMENT EXCITATION FROM WAVES

The moments and forces acting on the ship due to the waves are based on the Froude-Krylov Hypothesis which states that:

a. The presence of the ship does not disturb the pressure distribution in the waves.

b. The pressure distribution on the ship is determined by the hydrostatic pressure due to the position relative to the wave crest with a correction in order to take into account the decreasing orbital diameters of the water particles with increasing water depth.

Assuming a symetric ship with vertical frames at the waterline and following the development in reference (1) the lateral force, $\bar{y}$, and the yawing moment, $\bar{n}$, due to wave pressure are:

$$\bar{y} = F_{yy} \cdot LBT \cdot \rho g \; 2\pi \bar{h}/\lambda \; \sin(2\pi a/\lambda - w_e)$$

Having assumed the frequency of encounter is zero. $w_e = 0$

$$\bar{y} = F_{yy} \cdot LBT \cdot \rho g \; 2\pi \bar{h}/\lambda \; \sin(2\pi a/\lambda)$$

$$\bar{n} = F_{xx} \cdot \tfrac{1}{2}L \cdot LBT \cdot \rho g \; 2\pi \bar{h}/\lambda \; \cos(2\pi a/\lambda)$$

The coefficients $F_{yy}$ and $F_{xx}$ are:

$$F_{yy} = 2/\,LB \;\; \sin\gamma \int_{-L/2}^{L/2} f_w(x) \cos(2\pi x/\lambda)\, dx$$

$$F_{xx} = 4/L^2B \;\; \sin\gamma \int_{-L/2}^{+L/2} f_w(x) \sin(2\pi x/\lambda)\, dx$$

Because is not the true angle between wave direction and the centerline plane of the ship the equations must be modified as follows:

$$\text{True angle } \alpha = \gamma - \psi$$

$$\bar{y} = (F_{yy} + \Delta F_{yy}) \; LBT \cdot \rho g \; 2\pi \bar{h}/\lambda \; \sin(2\pi a/\lambda)$$

$$\bar{n} = (F_{xx} + \Delta F_{xx}) \; \tfrac{1}{2}L \cdot LBT \cdot \rho g \; 2\pi \bar{h}/\lambda \; \cos(2\pi a/\lambda)$$

$$\Delta F_{yy} = -\psi\, \partial F_{yy}/\partial\alpha$$

$$\Delta F_{xx} = -\psi\, \partial F_{xx}/\partial\alpha$$

$$\bar{y} = (F_{yy} - \psi\, \partial F_{yy}/\partial\alpha)\ LBT \cdot \rho g\ 2\pi \bar{h}/\lambda\ \sin(2\pi\, a/\lambda)$$

$$\bar{n} = (F_{xx} - \psi\, \partial F_{xx}/\partial\alpha)\ \tfrac{1}{2}L \cdot LBT \cdot \rho g\ 2\pi h/\lambda\ \cos(2\pi\, a/\lambda)$$

Non dimensionalizing the equations.

$$\bar{y}' = \bar{y}/\tfrac{1}{2}\ u^2\ S$$

$$\bar{n}' = \bar{n}/\tfrac{1}{2}\ u^2\ SL$$

Assuming that the ship velocity in the direction of the waves is the same as the wave velocity

wave speed $\quad c = \sqrt{g\lambda/2\pi}$

ship speed $\quad u = c/\cos\alpha$

$$u^2 = g\lambda/2\pi \cos^2\alpha$$

$$\bar{y}' = (F_{yy} - \psi\, \partial F_{yy}/\partial\alpha)\ \Phi \cos^2\alpha \sin(2\pi\, a/\lambda)$$

$$\bar{n}' = (F_{xx} - \psi\, \partial F_{xx}/\partial\alpha)\ \tfrac{1}{2}\ \Phi \cos^2\alpha \cos(2\pi\, a/\lambda)$$

$$\Phi = k/C_b\ \ m'\ 4\pi^2\ \bar{h}L/\lambda^2$$

k is the factor used to take into account the influence of the decreasing orbital velocities as shown in reference (1).

On the following page are the graphs showing the values of $F_{yy}$ and $F_{xx}$ coefficients for the Mariner hull.

Figure XV

Coefficient for Sway Exciting Force

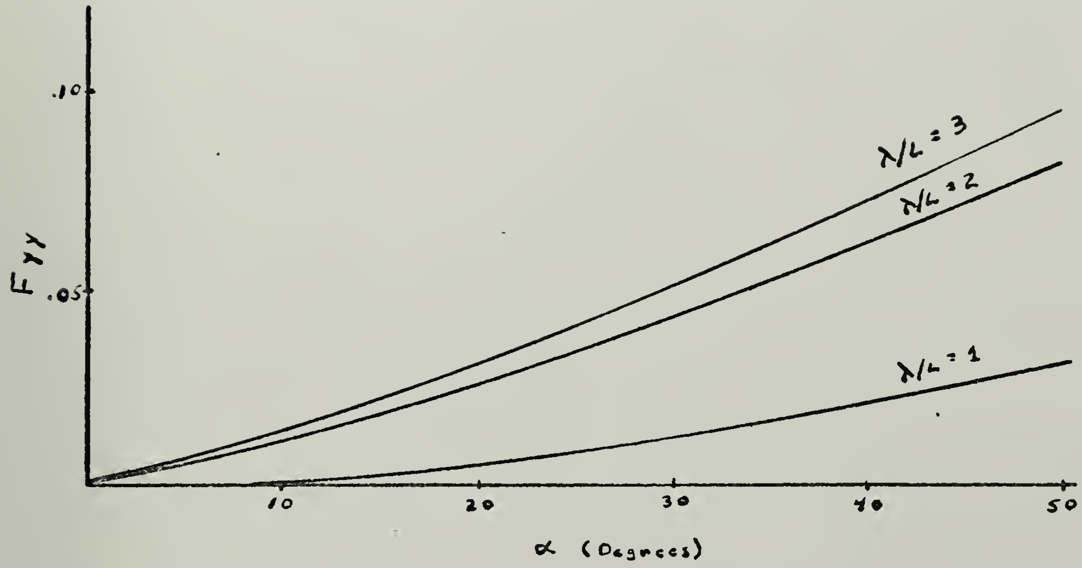



Figure XVI

Coefficient for Yaw Exciting Moment